Passport photos from the warehouse club take up to 30 minutes to be processed. Let X be the random variable **SCORE: _____ / 8 PTS**
representing the time (in **hours**) taken to process a randomly selected customer's passport photos. The probability density function is given by

$$f(x)=\begin{cases}\frac{k}{\sqrt{1-x^2}}, & x\in[0,\frac{1}{2}]\\ 0, & x\notin[0,\frac{1}{2}]\end{cases}$$ (for some appropriate constant k).

[a] Find the probability that a customer's photos take at least 10 minutes to be processed.

1 $\int_0^{\frac{1}{2}}\frac{k}{\sqrt{1-x^2}}\,dx=1$

$k\sin^{-1}x\Big|_0^{\frac{1}{2}}=1$

1 $k(\frac{\pi}{6}-0)=1$

$k=\frac{6}{\pi}$ $\frac{1}{2}$

$P(\frac{1}{6}\le X\le\frac{1}{2})=\int_{\frac{1}{6}}^{\frac{1}{2}}\frac{\frac{6}{\pi}}{\sqrt{1-x^2}}\,dx$ 1

$=\frac{6}{\pi}\sin^{-1}x\Big|_{\frac{1}{6}}^{\frac{1}{2}}$ $\frac{1}{2}$

$=\frac{6}{\pi}(\frac{\pi}{6}-\sin^{-1}\frac{1}{6})$

$=1-\frac{6}{\pi}\sin^{-1}\frac{1}{6}$ $\frac{1}{2}$

[b] Find the average/mean processing time in minutes.

1 $\int_0^{\frac{1}{2}}\frac{\frac{6}{\pi}x}{\sqrt{1-x^2}}\,dx$

$u=1-x^2$ $\begin{cases}x=\frac{1}{2}\rightarrow u=\frac{3}{4}\\ x=0\rightarrow u=1\end{cases}$

$du=-2x\,dx$

$-\frac{1}{2}du=x\,dx$

1 $-\frac{3}{\pi}\int_1^{\frac{3}{4}}u^{-\frac{1}{2}}\,du=-\frac{6}{\pi}u^{\frac{1}{2}}\Big|_1^{\frac{3}{4}}$ $\frac{1}{2}$

$=-\frac{6}{\pi}(\frac{\sqrt{3}}{2}-1)$ $\frac{1}{2}$

$=\frac{3}{\pi}(2-\sqrt{3})$ HOURS

$=\frac{180}{\pi}(2-\sqrt{3})$ MINUTES $\frac{1}{2}$

For the function $f(x)=\sqrt{64-(x-2)^2}$ on $[-6,2]$, find the value of c such that $f_{ave}=f(c)$. **SCORE: _____ / 6 PTS**

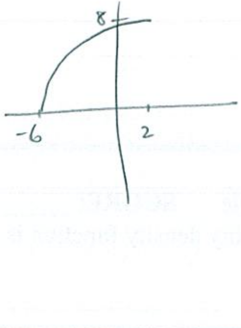
NOTE: This is the value c guaranteed by the Mean Value Theorem for Integrals.

$$f_{AVE}=\frac{1}{2-{-6}}\int_{-6}^{2}\sqrt{64-(x-2)^2}\,dx=\frac{1}{8}\cdot\frac{1}{4}\pi 8^2=2\pi \quad 1$$

$2\frac{1}{2}$

CENTER $(2,0)$

RADIUS 8



$$\sqrt{64-(x-2)^2}=2\pi \quad 1$$

$$64-(x-2)^2=4\pi^2$$

$$64-4\pi^2=(x-2)^2$$

$$\pm\sqrt{64-4\pi^2}=x-2$$

$$x=2\pm\sqrt{64-4\pi^2}$$

$$1\frac{1}{2} \quad x=2-2\sqrt{16-\pi^2}\in[-6,2]$$

Find the surface area if the curve $y = \frac{9}{2}e^{\frac{x}{3}} + \frac{1}{2}e^{-\frac{x}{3}}$ for $x \in [-6, 6]$ is revolved around the $x-$axis. **SCORE: _____ / 9 PTS**

$$2\pi \int_{-6}^{6} \left(\frac{9}{2}e^{\frac{1}{3}x} + \frac{1}{2}e^{-\frac{1}{3}x}\right)\sqrt{1 + \left(\frac{3}{2}e^{\frac{1}{3}x} - \frac{1}{6}e^{-\frac{1}{3}x}\right)^2}\, dx \quad 1$$

$$= 2\pi \int_{-6}^{6} \left(\frac{9}{2}e^{\frac{1}{3}x} + \frac{1}{2}e^{-\frac{1}{3}x}\right)\sqrt{1 + \frac{9}{4}e^{\frac{2}{3}x} - \frac{1}{2} + \frac{1}{36}e^{-\frac{2}{3}x}}\, dx$$

$$= 2\pi \int_{-6}^{6} \left(\frac{9}{2}e^{\frac{1}{3}x} + \frac{1}{2}e^{-\frac{1}{3}x}\right)\sqrt{\frac{9}{4}e^{\frac{2}{3}x} + \frac{1}{2} + \frac{1}{36}e^{-\frac{2}{3}x}}\, dx \quad 1$$

$$= 2\pi \int_{-6}^{6} \left(\frac{9}{2}e^{\frac{1}{3}x} + \frac{1}{2}e^{-\frac{1}{3}x}\right)\left(\frac{3}{2}e^{\frac{1}{3}x} + \frac{1}{6}e^{-\frac{1}{3}x}\right) dx \quad 1 \quad 1$$

$$= 2\pi \int_{-6}^{6} \left(\frac{27}{4}e^{\frac{2}{3}x} + \frac{3}{2} + \frac{1}{12}e^{-\frac{2}{3}x}\right) dx \quad 1$$

$$= 2\pi \left(\frac{81}{8}e^{\frac{2}{3}x} + \frac{3}{2}x - \frac{1}{8}e^{-\frac{2}{3}x}\right)\Big|_{-6}^{6} \quad 1$$

$$1 \quad = 2\pi \left(\frac{81}{8}(e^{4} - e^{-4}) + \frac{3}{2}(12) - \frac{1}{8}(e^{-4} - e^{4})\right) \quad 1$$

$$= \pi\left(\frac{41}{2}(e^{4} - e^{-4}) + 36\right) \quad 1$$

Find the length of the curve $y = \frac{\sqrt{x}}{3}(3x-1)$ on the interval $x \in [1, 4]$. **SCORE: _____ / 7 PTS**

$$y = x^{\frac{3}{2}} - \frac{1}{3}x^{\frac{1}{2}}$$

$$\int_1^4 \sqrt{1 + \left(\underline{\tfrac{3}{2}x^{\frac{1}{2}} - \tfrac{1}{6}x^{-\frac{1}{2}}}\right)^2}\, dx \quad (1)$$

$$= \int_1^4 \sqrt{1 + \tfrac{9}{4}x - \tfrac{1}{2} + \tfrac{1}{36}x^{-1}}\, dx$$

$$= \int_1^4 \sqrt{\underline{\tfrac{9}{4}x + \tfrac{1}{2} + \tfrac{1}{36}x^{-1}}}\, dx \quad (2)$$

$$= \int_1^4 \underline{\left(\tfrac{3}{2}x^{\frac{1}{2}} + \tfrac{1}{6}x^{-\frac{1}{2}}\right)}\, dx \quad (1)$$

$$= \underline{\left(x^{\frac{3}{2}} + \tfrac{1}{3}x^{\frac{1}{2}}\right)}\Big|_1^4 = \underline{(8-1) + \tfrac{1}{3}(2-1)} = \underline{7\tfrac{1}{3}}$$

$$(1) \qquad (1) \qquad (1)$$